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Spin Structure of the Proton

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ABSTRACT

By assuming that there is no significant intrinsic polarization of the gluon, we have computed the polarized quark contributions to the proton's spin under SU(3) flavor symmetry breaking for the polarized sea and have performed a global leading-order QCD fit to obtain the spin-dependent quark distributions, which could be used as input for analyzing lepton-hadron and hadron-hadron collisions.

The measurement of spin-related observables in processes involving polarized protons provides to probe correlation between the proton's spin and the spins of its constituents. There have been several investigations of polarized parton distributions inspired by the EMC measurement [1] of the polarized structure function g_1^p . Together with the latest available data, the recent SMC measurement [2] of the integrated proton structure function g_1^p is

$$\int_0^1 dx g_1^p(x) \text{ (SMC/EMC/SLAC) } = 0.142 \pm 0.008 \pm 0.011 \text{ (} Q^2 = 10 \text{ GeV}^2 \text{)}. \quad (1)$$

In the naïve parton model, g_1^p is related to the polarized quark densities $(q_i^\uparrow, q_i^\downarrow)$ with spin parallel or antiparallel to the longitudinally polarized parent proton (at momentum transfer scale Q^2)

$$g_1^p(x, Q^2) = \frac{1}{2} \sum_q e_q^2 [\Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2)], \quad (2)$$

where e_q 's are the quark charges and $\Delta q(x, Q^2) = q^\uparrow(x, Q^2) - q^\downarrow(x, Q^2)$. When combined with the values of the parameters F and D determined from low-energy neutron and hyperon beta decays by means of SU(3) flavor symmetry, the first moment of the flavor singlet part of g_1^p is very close to zero. This implies that the total fraction of the proton helicity carried by all quarks and antiquarks almost vanishes, in contradiction to previous naïve expectation [3]. Suggestions have been made to explain this startling conclusion in terms of a large negative polarization of the sea quarks inside the proton [4] or a

large positive polarization for the gluons [5] or a suitable combination of both [6].

However, the result of recent NMC measurement [7] hints that the u - d flavor symmetry of the sea is broken. This hint comes from the violation of the Gottfried sum rule. The NMC obtained the Gottfried sum to be $S_G = 0.235 \pm 0.026$ ($Q^2 = 4\text{GeV}^2$). Under the assumptions of isospin symmetry for the nucleon and for the sea quark distributions in the proton, the Gottfried sum reads

$$\begin{aligned} S_G &= \int_0^1 \frac{dx}{x} [F_2^{\mu p}(x) - F_2^{\mu n}(x)] \\ &= \frac{1}{3} \int_0^1 dx [u(x) + \bar{u}(x) - d(x) - \bar{d}(x)] \\ &= \frac{1}{3}. \end{aligned} \tag{3}$$

This implies a large SU(2) flavor symmetry breaking in the quark sea or the suppression for the production of $u\bar{u}$ pairs relative to $d\bar{d}$ pairs in the proton:

$$\begin{aligned} d_s - u_s &= \bar{d} - \bar{u} \\ &= \int_0^1 dx [\bar{d}(x) - \bar{u}(x)] \\ &= 0.147. \end{aligned} \tag{4}$$

In this paper, we aim at understanding the spin content of the polarized sea quarks and providing an ingredient in decoding the spin puzzle.

Let us consider the first moment of the spin-dependent structure function in the parton model with QCD corrections. It is given entirely by the proton

matrix elements a_0, a_3 and a_8 of the axial vector currents multiplied by the relevant Wilson coefficients [8]

$$\begin{aligned} \int_0^1 dx g_1^p(x, Q^2) &= \frac{1}{12} \left(1 - \frac{\alpha_s(Q^2)}{\pi}\right) a_3 + \frac{1}{36} \left(1 - \frac{\alpha_s(Q^2)}{\pi}\right) a_8 \\ &+ \frac{1}{9} \left(1 - \frac{\alpha_s(Q^2)}{\pi} \left(\frac{33 - 8N_f}{33 - 2N_f}\right)\right) a_0 \\ &- \frac{\alpha_s(Q^2)}{6\pi} \Delta G(x, Q^2), \end{aligned} \quad (5)$$

where the gluon contributes to g_1^p via the γ_5 -triangle anomaly [5, 9]. The matrix elements can be decomposed into non-singlet and singlet polarized quark distributions. In terms of quark polarizations $\Delta q \equiv \int dx (q^\uparrow(x) - q^\downarrow(x))$, or the F -type and D -type couplings, the matrix elements read

$$\begin{aligned} a_3 &= \Delta u + \Delta \bar{u} - \Delta d - \Delta \bar{d} \equiv F + D, \\ a_8 &= \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} - 2\Delta s - 2\Delta \bar{s} \equiv 3F - D, \\ a_0 &= \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} \equiv 2S_z^{quarks}, \end{aligned} \quad (6)$$

where the singlet part a_0 corresponds the total spin carried by all the quarks.

We assume that there is no significant intrinsic polarization of the gluon ($\Delta G = 0$). Equating (5) to (1) and taking $N_f = 3$ and $\Lambda_{QCD} = 177 MeV$ for (5) at the starting point of our perturbative evolution $Q^2 = 4 GeV^2$, we obtain

$$\begin{aligned} \Delta u_v &= 2F + 2\Delta s - 2\Delta u_s = 0.825 - 2\Delta u_s, \\ \Delta d_v &= F - D + 2\Delta s - 2\Delta d_s = -0.432 - 2\Delta d_s, \\ \Delta s &= -0.047 \pm 0.025, \end{aligned} \quad (7)$$

where $\Delta\bar{u} = \Delta u_s$, $\Delta\bar{d} = \Delta d_s$ and $\Delta\bar{s} = \Delta s$ are assumed and might be considered since there is also a contribution to the proton's spin from the orbital angular momentum of the partons. In (7) we have used $F = 0.459 \pm 0.008$ and $D = 0.798 \mp 0.008$ from ref.[10]. The general spin decomposition of the proton is (in the Skyrme model [4, 11] and its simple extensions, there are no gluons ($\Delta G = 0$))

$$\frac{1}{2} = S_z^{quarks} + \Delta G(=0) + \langle L_z \rangle . \quad (8)$$

The compensation between the quark helicities and the orbital angular momentum to the proton in the infinite momentum frame is consistent with QCD in such a way that the proton's spin is unchanged. We assume that the polarized sea distributions for u, d and s quarks are different. To a good approximation, the Pauli exclusion principle guides us in making reasonable assumptions for various polarized parton distributions. Using (7) to relate the valence distributions to the values of the axial coupling at $Q^2 = 4GeV^2$, we have

$$\begin{aligned} u_v^\uparrow &= 1 + F + \delta , & u_v^\downarrow &= 1 - F - \delta , \\ d_v^\uparrow &= \frac{1 + F - D}{2} + \beta , & d_v^\downarrow &= \frac{1 - F + D}{2} - \beta , \end{aligned} \quad (9)$$

where

$$\begin{aligned} \delta &= \Delta s - \Delta u_s , \\ \beta &= \Delta s - \Delta d_s . \end{aligned} \quad (10)$$

When $Q^2 \rightarrow 0 \text{ GeV}^2$, no contribution from the sea ($\delta = \beta = 0$), one has $\Delta u_v = 2F$ and $\Delta d_v = F - D$ [12]. The Pauli principle might be advocated to account for the distribution of the u quark with respect to that of the d quark in the proton. Therefore, we rewrite u and d in (9) as

$$\begin{aligned} u_v^\downarrow &= (1 - F - \delta)d_v, \\ d_v^\downarrow &= \left(\frac{1 - F + D}{2} - \beta\right)\frac{u_v}{2}, \end{aligned} \quad (11)$$

and generalize the above relations to the sea quark distributions

$$\begin{aligned} u_s^\downarrow &= (1 - F - \delta)d_s, \\ d_s^\downarrow &= \left(\frac{1 - F + D}{2} - \beta\right)\frac{u_s}{2}. \end{aligned} \quad (12)$$

We first start from $\Delta G = 0$ and consider an ansatz for spin-dependent quark distributions based on SU(3) flavor symmetry breaking effect for polarized sea. From (4), (10) and (12), we obtain

$$\begin{aligned} \Delta u_s &= \Delta \bar{u} \\ &= u_s - 2u_s^\downarrow \\ &= \frac{-0.147 + (-1 + 2F + 2\Delta s)d_s}{1 + 2d_s}, \end{aligned} \quad (13)$$

and, similarly,

$$\begin{aligned} \Delta d_s &= \Delta \bar{d} \\ &= \frac{0.294 + (1 + F - D + 2\Delta s)u_s}{2 + 2u_s}, \end{aligned} \quad (14)$$

where $\bar{u}^\uparrow = u_s^\uparrow$, $\bar{u}^\downarrow = u_s^\downarrow$, $\bar{d}^\uparrow = d_s^\uparrow$ and $\bar{d}^\downarrow = d_s^\downarrow$ are assumed.

In the following analysis we shall use the set MRS(A) of unpolarized parton distribution functions given in ref. [13]. This set is extracted at the reference scale $Q^2 = 4\text{GeV}^2$ from several experiments. The values of d_s and u_s can be obtained by the integrated MRS(A) set with the distributions of the c quarks in the proton being set identically to zero. From (7), (13) and (14), the normalizations of the polarized sea-quark distributions are given by

$$\begin{aligned}\Delta u_s &= -0.093, & \Delta d_s &= 0.262, \\ \Delta u_v &= 1.011, & \Delta d_v &= -0.956.\end{aligned}\tag{15}$$

Including QCD corrections, we get $\Delta d_s \approx -2.8\Delta u_s \approx -5.6\Delta s$ and $S_z^{quarks} = 0.149 \pm 0.037$ and most of the proton's spin come from the orbital angular momentum L_z . A nonzero L_z has the phenomenological consequence that there is an intrinsic transverse momentum, since in the naïve parton model with all partonic momentum parrallel to the parent's momentum, one has $L_z = 0$. Contrary to SU(3) invariance of sea polarization, (15) implies that d_s quarks prefer to polarize in the same direction of the proton's spin, while u_s and s quarks do not. Perturbative QCD arguments [14] suggest that the valence quarks at $x = 1$ remember the spin of the parent proton, and the quark distributions are dominated by the sea as $x \rightarrow 0$. We shall take the following form for polarized valence-quark distributions [15]:

$$\Delta q_v(x) = \left(\frac{x - x_0}{1 - x_0} \right) x^p q_v(x),\tag{16}$$

which satisfies the boundary conditions and x_0 is a free parameter. The sign of $\Delta q_v(x)$ is flipped at $x = x_0$ when p is positive. Since Δu_v is positive, we take $x_0 = 0$ and write the parametrization for Δu_v as $\Delta u_v(x) = x^{\gamma_{u_v}} u_v(x)$. While for Δd_v , we have $\Delta d_v < 0$, the helicity of a d_v quark relative to its parent proton is flipped. Using the normalizations (15), we obtain the parametrizations

$$\begin{aligned}\Delta u_v(x) &= x^{0.275} u_v(x), \\ \Delta d_v(x) &= \left(\frac{x - 0.9}{1 - 0.9} \right) x^{0.818} d_v(x).\end{aligned}\tag{17}$$

In order to ensure that the positivity constraints ($|\Delta q_s(x)| \leq q_s(x)$) be satisfied for $0 \leq x \leq 1$, we use the simple parametrizations $\Delta q_s(x) = \pm x^{\gamma_{q_s}} q_s(x)$, where the “+” sign is for $\Delta q_s > 0$ and the “−” sign is for $\Delta q_s < 0$. The γ_{q_s} ’s are given by

$$\begin{aligned}\gamma_{u_s} &= 0.693, \quad \gamma_{d_s} = 0.521, \\ \gamma_s &= 0.722.\end{aligned}\tag{18}$$

One observes that one can use the general parametrization form $\Delta q_s(x) \approx x^m (1-x)^n q_s(x)$, but this does not make any practically different distributions since the behavior of sea quarks is considered to be dominated at small x .

All quark spin distributions are determined initially at $Q^2 = 4\text{GeV}^2$ and are used as the input distributions, from which we obtain the distributions at higher Q^2 values by a numerical integration [16] of the Altarelli-Parisi

(AP) equations for the polarized case [17]. With three different Q^2 values, the theoretical fits to the data on $xg_1^p(x)$ from the EMC and SMC [1, 2] are shown in Fig. 1. Fig. 1 shows that the Q^2 variation has small effect on $xg_1^p(x)$. Fig. 2 exhibits the Q^2 evolution of the polarized sea-quark densities. The distributions $\Delta u_s(x)$ and $\Delta s(x)$ increase while $\Delta d_s(x)$ decreases with Q^2 . In Fig. 2, $\Delta d_s(x)$ drastically differs from $\Delta u_s(x)$ and $\Delta s(x)$. We may regard the polarized quark distributions as having the Pauli exclusion principle reflected by the SU(3) flavor symmetry breaking for the sea polarization. Likewise, shown in Fig. 3 is the x behavior of the spin-dependent gluon densities, which are dynamically generated from the bremsstrahlung of the quarks. These different- Q^2 behaviors indicate that if the gluon is initially unpolarized at the perturbative evolution scale then the induced gluon polarization will be generated at larger Q^2 . The gluon spin does increase with $\ln Q^2$ in Fig. 4, but the quark-induced ΔG is small. The resulting ΔG is sensitive to the initial shape of the gluon distribution, which gives a contribution to g_1^p . Fig. 5 shows that the net quark polarization is nearly independent of Q^2 .

In this paper we have obtained a consistent set of spin-dependent quark distributions which provide a tool for deep inelastic structure function measurements and theoretical calculations of cross sections in polarized hadron-hadron and lepton-hadron collisions. One may wish to consider a non-zero polarized charm sea for $Q^2 > m_c^2$ and to make a change of basis from a_3 , a_8 and a_0 to a_3 , a_8 , $a_{15}(= \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} - 3\Delta c - 3\Delta \bar{c})$

and $a_0(= \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} + \Delta c + \Delta \bar{c})$ at the initial Q^2 . In practice, the non-perturbative contribution of the polarized c-quark densities is too small to distort the analysis at large x , but it does modify the normalization and the shape of the polarized parton distributions in the small x region. It is also interesting to note that there are $\bar{s}s$ pairs in the proton due to $\Delta s \neq 0$. One can extend the analysis to dynamical processes which involve OZI-forbidden diagrams. Phenomenologically, the spin-dependent parton densities improve the description of the polarized nuclear structure functions but we need more accurate experimental data on the nucleon spin to understand the spin-transfer mechanism and to clarify how the proton's spin is distributed among its valence quarks and the flavored $q\bar{q}$ sea.

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Figure Captions

Figure 1. Fits to the $g_1^p(x)$ structure function at three values of Q^2 .

Figure 2. The evolution of polarized sea densities at three values of Q^2 .

Figure 3. The evolution of polarized gluon densities at three values of Q^2 .

Figure 4. The x -integrated distribution ΔG over the range $10^{-4} < x < 1$.

Figure 5. The x -integrated distribution a_0 over the range $10^{-4} < x < 1$.

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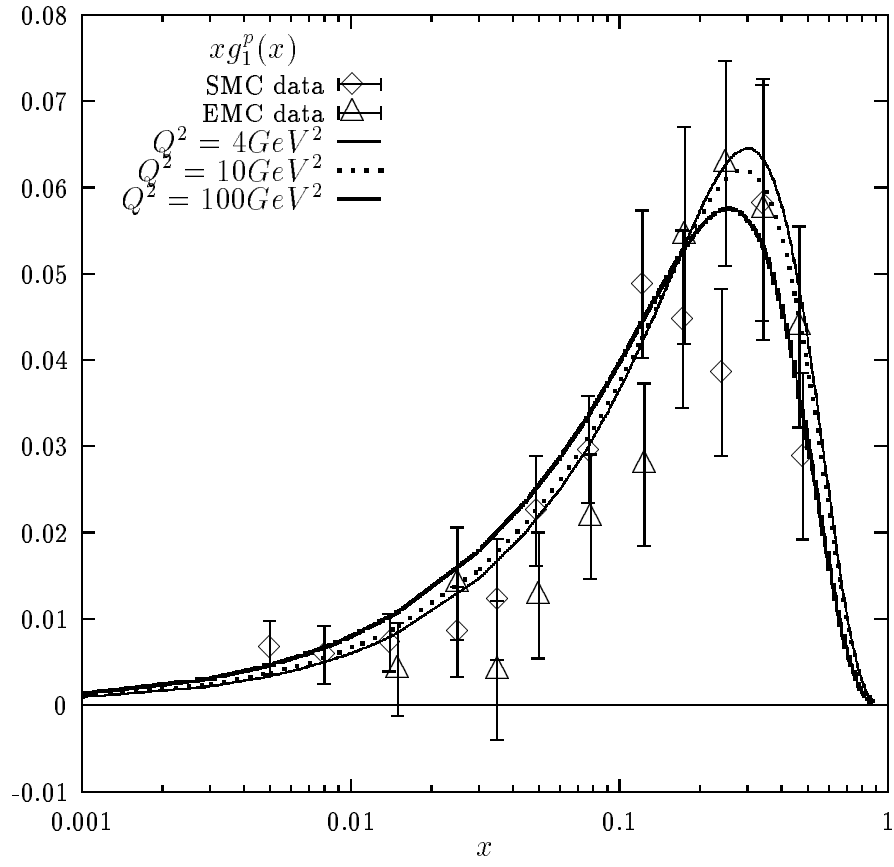


Figure 1: Fits to the $g_1^p(x)$ structure function at three values of Q^2

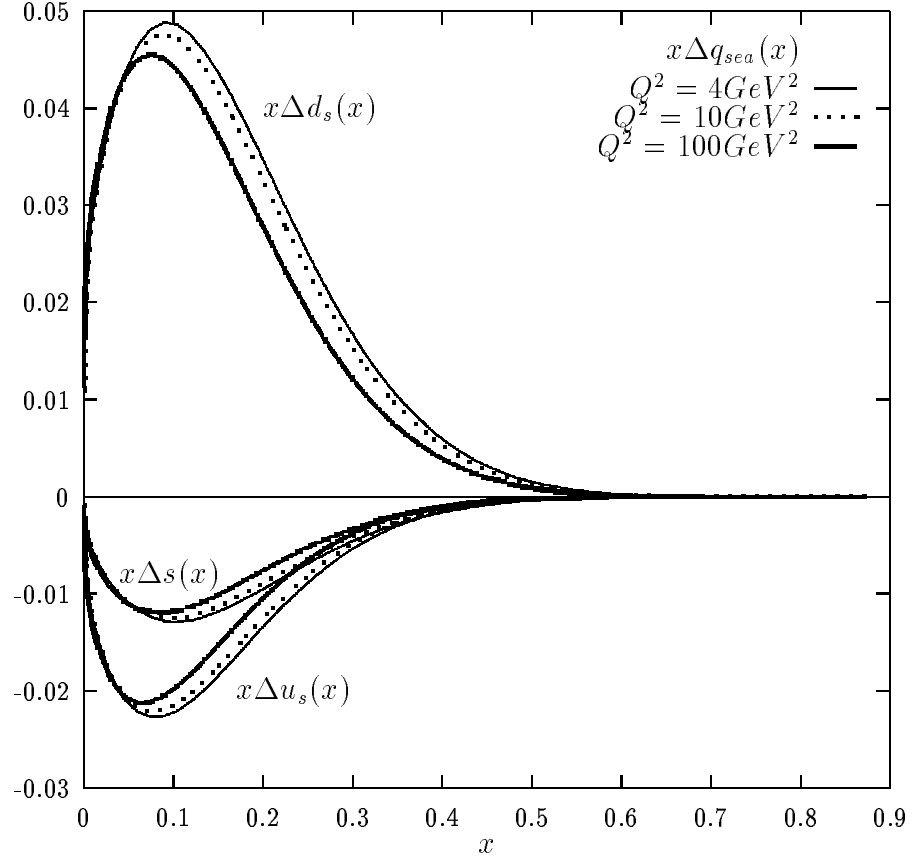


Figure 2: The evolution of polarized sea densities at three values of Q^2

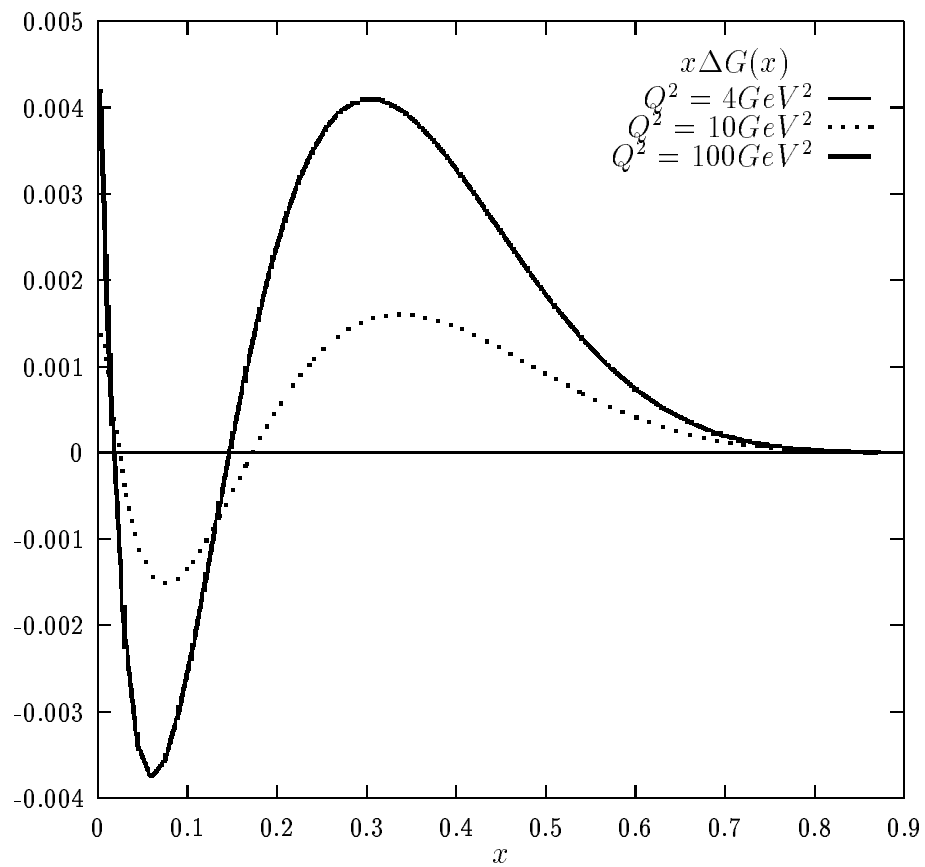


Figure 3: The evolution of polarized gluon densities at three values of Q^2

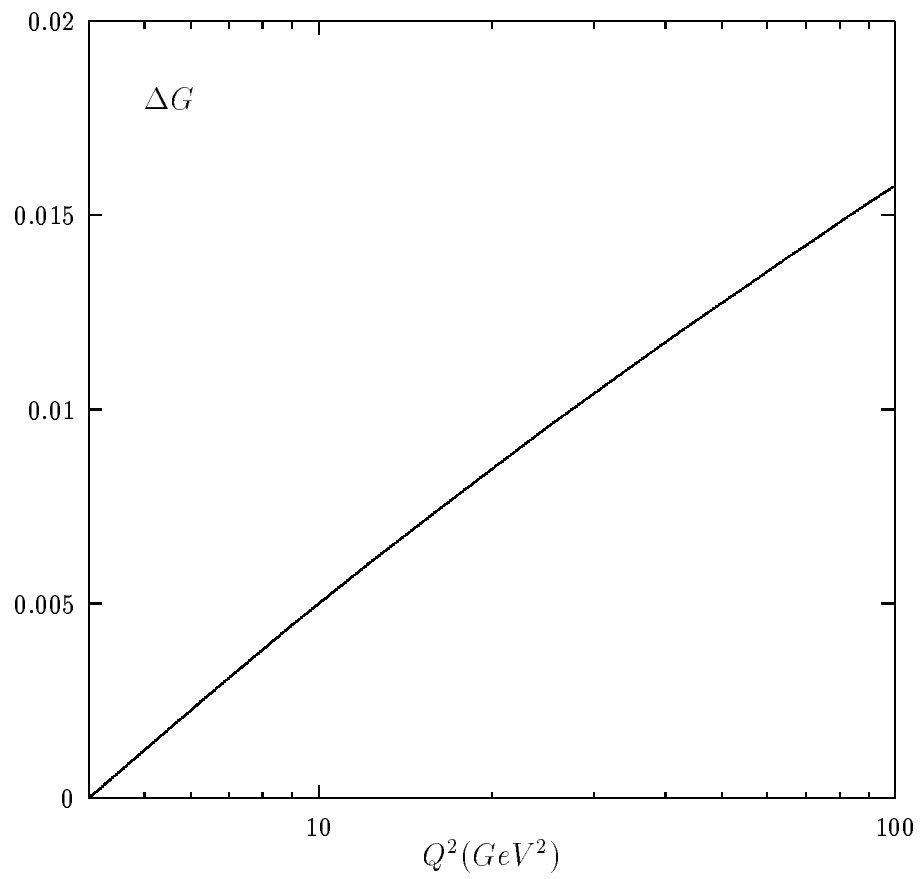


Figure 4: The x -integrated distribution ΔG over the range $10^{-4} < x < 1$

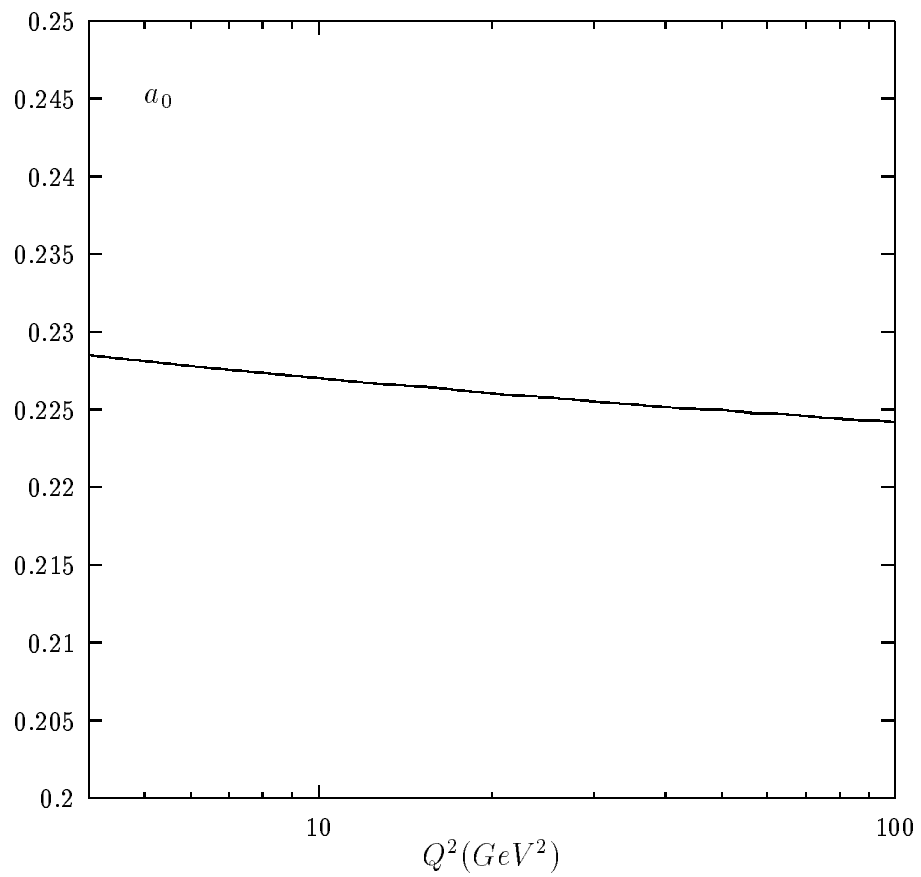


Figure 5: The x -integrated distribution a_0 over the range $10^{-4} < x < 1$